Motivation

In this thesis I discuss the properties of Snyder space, which is a formulation of quantum mechanics on a quantized spacetime. Snyder space holds an important historical place since it is the first formulation of quantum mechanics on a Noncommutative manifold. There has been much work recently studying consequences of formulating quantum mechanics on noncommutative manifolds, and it has become commonplace to cite Snyder space for its historical precedence. However, very little has actually been done to study the properties of Snyder space. In this work I seek to alleviate this problem by presenting a systematic overview of Snyder space and compare its results to similar results found in the literature for other noncommutative spaces.

The question I am addressing is open-ended. I have been influenced to a small extent by the results that can be found in the literature for other noncommutative spaces. Partially for this reason I chose to study the simple harmonic oscillator in Snyder space, which has proven to be a fruitful subject in other spaces. However, this research has primarily been an exploratory process; I did not know a priori what the results would be, just as Lewis and Clark did not know exactly what they would find on their journey to the Pacific Ocean. This exploration is characteristic of theoretical physics in general. At present there is no significant discrepancy between theory and experimental results in high-energy physics. In spite of this agreement, there is some dissatisfaction with certain aspects of the theory. In particular, the two major theories of theoretical physics, quantum field theory and general relativity, are apparently incompatible. A major goal is to unify these theories into a single unified theory. As such, theoretical physicists make conjectures about possible alterations that could be made to a theory that could lead to unification, and then catalog results that experimentalists can seek to verify or to annul. Some proposed alterations are taken more seriously than others, and some conjectures are considered by mathematical physicists solely as mathematical exercises.

I chose to study Snyder space for several reasons. First, as I already mentioned, it holds a significant place in the history of physics as being the first formulation of quantum mechanics on a noncommutative manifold, but it has not been seriously studied since its original conception in the 1940s. Secondly, Snyder space was originally proposed to solve a problem in quantum field theory, and while it was never shown to be deficient in solving this problem, it has become obvious that it is closely related to proposed theories of quantum gravity, far removed from its original context. Also, the commutation algebra of Snyder space has been derived by methods independent of Snyder's original derivation, suggesting that it could be a viable model of nature. Finally, a formulation of quantized spacetime is an interesting exercise in mathematical physics, the results of which should be cataloged and investigated.

In this first chapter, I elaborate on each of these reasons for reconsidering Snyder space and outline the structure of the thesis as a whole.