Chapter 3: The Gravitational Interaction

Review

- Use Newton's first law to explain why you should drive slowly in ice and snow.

The reluctant horse

- "Every time I pull on the cart, it pulls back with equal force. Therefore I cannot pull the cart."

Understanding the gravitational force: A plan of action

- We will discover some things about gravity by studying the motion of objects.
- We will assume that Newton's second law \( F = ma \) is true, and ask what kind of force is necessary to explain the accelerations that we observe due to earth's gravity.
- We will assume that gravity depends only on the mass of the object and its distance from the center of the earth.

Vocabulary: Mass vs. Weight

- Mass is the property of an object that resists acceleration.
  - The metric unit of mass is the kilogram
  - What is the unit of mass in the English system of units (used in the U.S.)?
- Weight is the force of gravity on an object.
- Mass and weight are related, but they are not the same thing!
What direction does the force of gravity point?

Decomposing motion into its components

First let’s consider distance for everyday interactions

Conclusion: For everyday interactions, the gravitational distance to the center of the earth is roughly constant, so the force (i.e., the weight) is constant

A free falling cow in the absence of air resistance

“Michelle” the cow in Manson Washington

Will a water droplet fall faster or slower than a cow?

\[ \text{Velocity} = \text{Distance} / \text{Time} \]

\[ \text{Acceleration} = \text{Change in Velocity} / \text{Change in Time} \]

\[ \text{Distance} = \text{Velocity} \times \text{Time} \]

\[ \text{Force of Gravity} = \text{Mass} \times \text{Acceleration} \]

<table>
<thead>
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<th>Distance (feet)</th>
<th>Velocity (ft/sec)</th>
<th>Acceleration (ft/sec^2)</th>
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Is anyone epileptic?

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<th>Acceleration (m/s^2)</th>
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<tr>
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If I drop a book and a piece of paper, which one will hit the table first?

A. The book  
B. The paper  
C. Neither, they hit the same time

Gravity causes a uniform acceleration at the planet’s surface (independent of mass)

But Newton’s 2nd law said

\[
\text{Acceleration} = \frac{\text{Force}}{\text{Mass}}
\]

The force of gravity must be proportional to the mass of an object!

If objects are gravitationally attracted to each other, why doesn’t the moon fall to the earth?

The moon is falling! (Dependence of the force of gravity on distance)

\[
\frac{\text{Force in orbit}}{(\text{mass}) \times (\text{acceleration in orbit})} = \frac{1}{3600}
\]

\[
\frac{\text{Distance to the moon}}{\text{Distance to the surface}} = \frac{1}{60}
\]

When Newton compared these two measurements, he concluded that the force of gravity is inversely proportional to the distance squared.

Gravity

- The equation for gravity is

\[
F = G \frac{Mm}{d^2}
\]

F = force of gravity
M = mass of the larger object
m = mass of the smaller object

G is just a small number
The gravitational acceleration of any body is independent of its own mass:

\[ a = \frac{F}{m} = \frac{GMm}{d^2m} = \frac{GM}{d^2} = g \]

- More massive objects feel a greater force of gravity but resist accelerating by the same amount.
- Heavy and light things all fall at the same rate.
- You can calculate the weight simply as \( W = mg \)

\[ W = mg \]

If \( d_2 \) is twice as large as \( d_1 \), how is the force different for the \( d_2 \) case?

A. The force is four times bigger
B. The force is twice as much
C. The force is half as much
D. The force is one quarter as much

\[ F = \frac{G Mm}{d^2} \]

If \( m_2 \) is twice as large as \( m_1 \), How is the force different (\( d \) is the same in both cases)?

A. The force is the same
B. The force is twice as much
C. The force is half as much
D. Need more information to tell

\[ F = \frac{G Mm}{d^2} \]
Two types of mass: Inertial and gravitational

\[ F = ma \quad F = G \frac{Mm}{d^2} \]

The ship is sailing to the left. The ball is attached to the mast and then dropped. What will happen? (neglect air resistance)

A. It will fall straight along the mast
B. It will fall straight down towards the point above which it was dropped (the sailor might want to duck)
C. It will fall in front of the mast, so the sailor is safe

The first cartoon illustrates what people in Aristotle's time (incorrectly) thought would happen. Galileo correctly predicted what really happens (shown in the live action).
The cow, the Russian cargo plane, and the Japanese fishing trawler. (Courtesy of Dave Barry)